Pergamon

Int. J. Heat Mass Transfer. Vol. 39, No. 13, pp. 2745–2750, 1996

0017–9310/96 \$15.00+0.00

0017-9310(95)00357-6

Mean fluid temperatures in direct contact heat exchangers without phase change

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(*Received* 2 *February* 1995 *and in final form* 10 *August* 1995)

Abstract—This paper deals with the distribution of fluid temperatures in a direct contact heat exchanger. The exchanger is considered as a column with two immiscible fluids in countercurrent flow. None of the fluids undergoes a phase change so that only sensible heat is transferred. Equations for the mean temperatures of the fluids are derived analytically by integration of the one-dimensional energy equations. The influences of decisive parameters on the temperature distributions along the column are examined. Comparisons with experiments show a satisfactory agreement between calculated and measured fluid temperatures. Copyright © 1996 Elsevier Science Ltd.

INTRODUCTION AND PROBLEM FORMULATION

As a result of the complex hydrodynamics, the heat transfer between immiscible fluids in direct contact heat exchangers cannot be predicted with satisfying accuracy. In such heat exchangers one phase is usually dispersed as bubbles or droplets, which move irregularly in the other phase so that the hydrodynamic processes follow generally statistical laws. Approximate considerations assume the processes to be one-dimensional and steady-state.

Based on these assumptions, simple differential equations for the mean fluid temperatures have been obtained from energy balances. In the case of a countercurrent flow arrangement, Fig. 1, the equations are given by Battya *et al.* [1] as

$$\kappa_c \rho_c c_{pc} \frac{d^2 T_c}{dz^2} + \frac{\rho_c \dot{V}_c c_{pc}}{A} \frac{dT_c}{dz} = \alpha_v (T_c - T_d) \qquad (1)$$

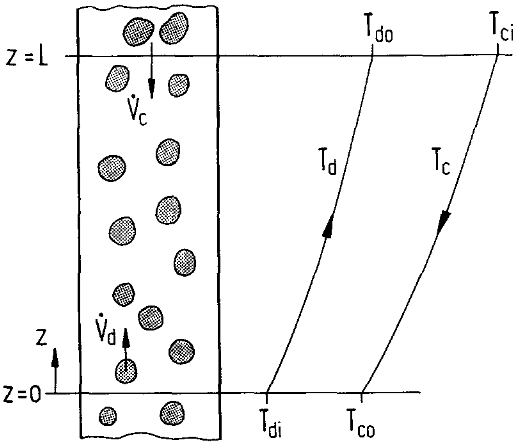

Fig. 1. Schematic of temperature distributions along a column.

$$\frac{\rho_d \dot{V}_d c_{pd}}{A} \frac{dT_d}{dz} = \alpha_v (T_c - T_d). \qquad (2)$$

Here T denotes the mean fluid temperature in the cross-section A of the column, z the axial coordinate, ρ the fluid density, c_p the specific heat capacity, $\dot{V}$ the volume flow rate, α_v the volumetric overall heat transfer coefficient and κ the axial dispersion coefficient. The subscripts c and d denote the variables pertinent to the continuous and dispersed phase, respectively.

Equations (1) and (2) have been integrated numerically by Battya *et al.* [1] using the boundary conditions

$$z = 0 : T_c = T_{co} \quad \frac{dT_c}{dz} = 0 \quad \text{and} \quad T_d = T_{di}. \qquad (3)$$

The authors compared their numerical results with experimental data from the literature and found a satisfactory agreement at larger values of z. At the smaller values of z, however, the calculated temperature profiles of the continuous phase deviate from the measured. Whereas the experiments lead to temperature profiles with considerable slopes at $z = 0$, the gradients of the numerically obtained temperature distributions are zero there because of the chosen boundary condition ($z = 0 : dT_c/dz = 0$).

Shchuplyak *et al.* [2] obtained an analytical equation for the temperature T_c of the continuous phase. According to the boundary conditions used, the temperature change of the continuous phase is zero at the end of the column, that is, $z = L : dT_c/dz = 0$, which is basically the same as specified in equation (3), because the position $z = L$ in ref. [2] is identical with the position $z = 0$ in ref. [1].

A comment seems to be appropriate concerning the temperature gradient $dT_c/dz = 0$ as the boundary

NOMENCLATURE

A	cross-sectional area of column
B	parameter, equation (7)
C	parameter, equation (7)
c_p	specific heat capacity
D	column diameter
G	parameter, equation (7)
K_0, K_1, K_2	constants of integration, equations (18)–(20)
L	column height
M_1, M_2	parameters, equation (21)
N_1, N_2	parameters, equation (22)
T	temperature
$\dot{V}$	volume flow rate.

Greek symbols

α	overall heat transfer coefficient
λ	root of characteristic equation
κ	axial dispersion coefficient
θ	dimensionless temperature difference, continuous phase
η	dimensionless axial coordinate
φ	dimensionless temperature difference, disperse phase.

Subscripts

c	continuous phase
d	disperse phase
v	volumetric
i	inlet
o	outlet.

condition†. Such a boundary condition, certainly convenient for a mathematical treatment of the problem, can hardly represent the physical reality in a general case when the fluids possess different temperatures ($T_c \neq T_d$) at any position z along the column and heat is transferred between them.

To underline our doubts, we consider a stationary fluid element exposed to the same environment as it exists along a flow path of this element through the heat exchanger. As long as the temperature of the environment is different from that of the fluid element, the temperature T of the fluid element changes with time t and the derivative $\mathrm{d}T/\mathrm{d}t$ is different from zero. If we now, at a certain time $t = \tau$, suddenly interrupt any heat exchange with the environment, the derivative $\mathrm{d}T/\mathrm{d}t$ at this moment is still different from zero. For $t > \tau$ the derivative $\mathrm{d}T/\mathrm{d}t$ is zero, whereas the temperature T of the fluid element retains its value reached at the end of the exposure time.

The transition state of the stationary fluid element considered here can at any instant of the exposure time be attributed to the position of the fluid element which moves through the heat exchanger. The relation is given by the instantaneous velocity of the fluid element. If the flow in the exchanger is, for example, of piston-type with a constant velocity, the axial position of the fluid element along the flow path is directly proportional to the exposure time. The exposure time can, of course, be chosen so that the fluid element arrives at the column end. Then the heat transfer with the environment is suddenly interrupted and we have at the moment of interruption $\mathrm{d}T/\mathrm{d}t \neq 0$, respectively $\mathrm{d}T/\mathrm{d}z \neq 0$.

The main purpose of our paper, however, is to show that equations (1) and (2) can be integrated analytically. The coefficients in the equations are considered as independent of the axial position. The integration constants are determined using boundary conditions specified on the basis of the fluid temperatures. The analytically obtained temperature distributions are verified by comparison with experiments.

INTEGRATION OF THE DIFFERENTIAL EQUATIONS

For the sake of simplicity, we use the following transformations:

$$\eta = \frac{z}{L}, \quad \theta = \frac{T_c - T_{co}}{T_{co} - T_{di}} \quad \text{and} \quad \varphi = \frac{T_d - T_{di}}{T_{co} - T_{di}}. \tag{4}$$

With these relations, equations (1) and (2) take the forms

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}\eta^2} + B\frac{\mathrm{d}\theta}{\mathrm{d}\eta} - C(1+\theta) = -C\varphi \tag{5}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\eta} + G\varphi = G(1+\theta) \tag{6}$$

where B, C and G are nondimensional parameters given by

$$B = \frac{\dot{V}_c L}{\kappa_c A}, \quad C = \frac{\alpha_v L^2}{\kappa_c \rho_c c_{pc}}, \quad G = \frac{\alpha_v L A}{\rho_d \dot{V}_d c_{pd}}. \tag{7}$$

It should be noted that these parameters have already been introduced by Battya *et al.* [1]. They used, however, the diameter D of the column as the scaling length instead of the column height L, so that the parameters B, C and G as given above differ from those of Battya *et al.* In context with this, it should

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† A boundary condition of this kind has been used for the first time by Danckwerts [3] who considered the concentration change of a reacting fluid flowing through a bed of stationary solid particles.

also be mentioned that equations (5) and (6) do not agree with the corresponding equations in the paper of Battya *et al.* [1] due to the different definitions of the nondimensional temperature θ and φ.

In order to separate the functions θ and φ in equations (5) and (6) from each other, equation (5) is derived with respect to η, giving

$$\frac{d^3\theta}{d\eta^3}+B\frac{d^2\theta}{d\eta^2}-C\frac{d\theta}{d\eta}=-C\frac{d\varphi}{d\eta}. \tag{8}$$

Elimination of φ and $d\varphi/d\eta$ from equations (5), (6) and (8) leads to the following differential equation:

$$\frac{d^3\theta}{d\eta^3}+a_1\frac{d^2\theta}{d\eta^2}+a_0\frac{d\theta}{d\eta}=0 \tag{9}$$

with

$$a_0 = BG-C \tag{10}$$

$$a_1 = B+G. \tag{11}$$

Integration of equation (9) yields

$$\theta=\frac{K_0}{a_0}+K_1\,e^{\lambda_1\eta}+K_2\,e^{\lambda_2\eta} \tag{12}$$

where K_0, K_1 and K_2 are constants of integration; λ_1 and λ_2 are the roots of the characteristic equation and are determined from

$$\lambda_1=\frac{-a_1+\sqrt{a_1^2-4a_0}}{2} \tag{13}$$

$$\lambda_2=\frac{-a_1-\sqrt{a_1^2-4a_0}}{2}. \tag{14}$$

The function φ is obtained from equation (5) by inserting the function θ according to equation (12) and its derivatives. This leads to the expression

$$\varphi=1+\frac{K_0}{a_0}+K_1\left(1-\frac{\lambda_1^2+B\lambda_1}{C}\right)e^{\lambda_1\eta}+K_2\left(1-\frac{\lambda_2^2+B\lambda_2}{C}\right)e^{\lambda_2\eta}. \tag{15}$$

For the reasons explained above, the temperature gradient used by previous authors, see equation (3), will not be considered in the present paper as a boundary condition. This condition is replaced by the corresponding fluid temperature.

In practical applications, the temperatures of one of the fluids at the inlet and outlet of the heat exchanger are usually known. Since one temperature of the other fluid is also known, an energy balance leads to the missing fluid temperature†. The boundary conditions may thus be specified as follows (see Fig. 1):

$$\eta=\frac{z}{L}=0:\quad T_c=T_{co}\quad\text{and}\quad T_d=T_{di},$$

$$\text{that is }\theta=0\quad\text{and}\quad\varphi=0 \tag{16}$$

$$\eta=1:\quad T_d=T_{do},\quad\text{that is }\varphi=\varphi_o. \tag{17}$$

These conditions lead to the following expressions for the constants of integration:

$$K_0=-\frac{N_2-N_1-(\varphi_o-1)(M_2-M_1)}{M_1N_2-M_2N_1}a_0 \tag{18}$$

$$K_1=\frac{N_2-(\varphi_o-1)M_2}{M_1N_2-M_2N_1} \tag{19}$$

$$K_2=-\frac{N_1-(\varphi_o-1)M_1}{M_1N_2-M_2N_1}. \tag{20}$$

Here M_1, M_2, N_1 and N_2 are given as

$$M_1=\frac{\lambda_1^2+B\lambda_1}{C}\quad M_2=\frac{\lambda_2^2+B\lambda_2}{C} \tag{21}$$

$$N_1=(1-M_1)\,e^{\lambda_1}-1\quad N_2=(1-M_2)\,e^{\lambda_2}-1. \tag{22}$$

From equation (12) and (15), together with equations (18)–(22), the nondimensional temperatures θ and φ, as functions of the nondimensional axial coordinate η, can be evaluated.

DISCUSSION OF RESULTS

Using equations (12) and (15), the nondimensional temperatures θ and φ were calculated for numerous values of the parameters B, C and G. The calculations revealed a relatively weak influence of the parameters B and C on the fluid temperatures in comparison with that of the parameter G. Therefore, only the influences of the parameter G on the temperatures θ and φ are demonstrated in Fig. 2 for heating and in Fig. 3 for cooling of the disperse phase.

As follows from these Figures, the higher values of the parameter G lead to a stronger change of the temperature φ, particularly in the vicinity of the inlet, $\eta = 0$, of the disperse phase. An increase in G shifts the curves θ and φ closer together. Such a behaviour was expected because the parameters B and C, characterizing the continuous phase, remain constant. In this case, a variation of the parameter G is mainly associated with a variation of the flow rate of the disperse phase. Therefore, a higher flow rate of this phase leads to a stronger temperature change of the continuous phase.

The validity of the analytical solutions is proved by comparison of the calculated temperatures with the experiments from the literature, Figs 4–6. The experimental and numerical results, obtained by different authors [1, 4, 5], were taken from the paper of Battya

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† Possible temperature changes prior to mixing and after separation of the fluids must be taken into account in the determination of the temperatures used for boundary conditions. Such temperature changes depend generally on the exchanger design.

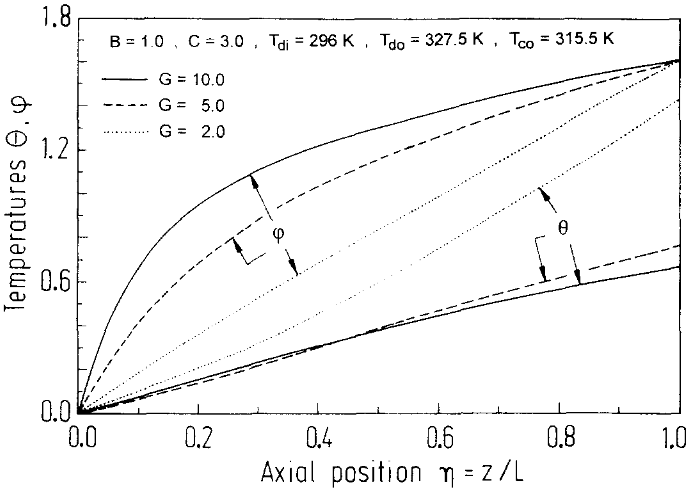

Fig. 2. Temperature changes along the column during heating of the disperse phase.

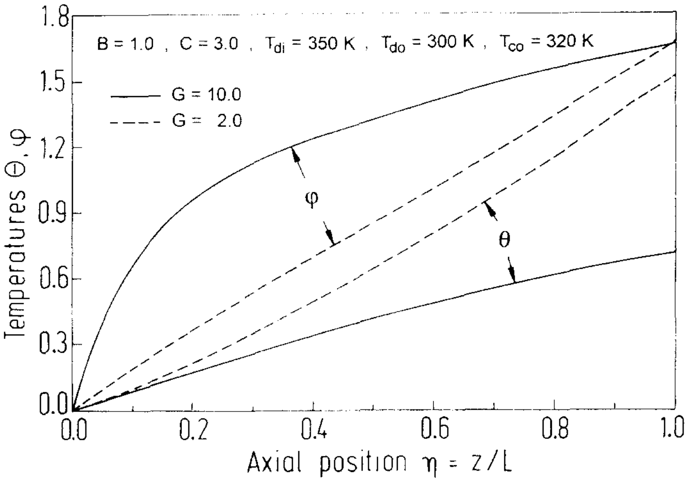

Fig. 3. Temperature changes along the column during cooling of the disperse phase.

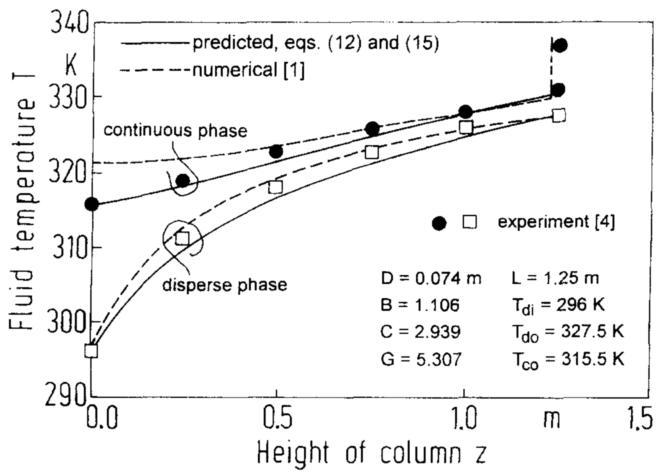

Fig. 4. Comparison of predicted temperatures with numerical [1] and experimental [4] data.

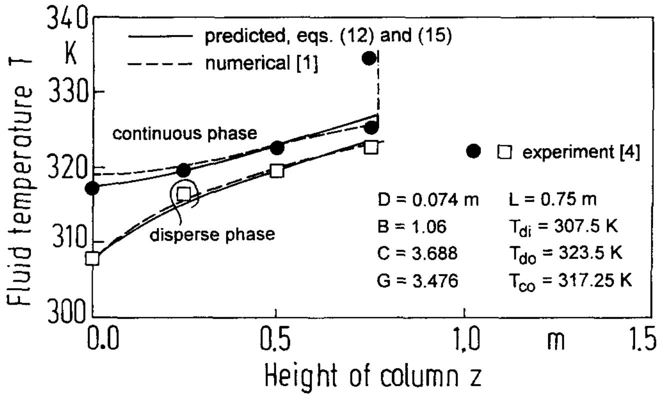

Fig. 5. Comparison of predicted temperatures with numerical [1] and experimental [4] data.

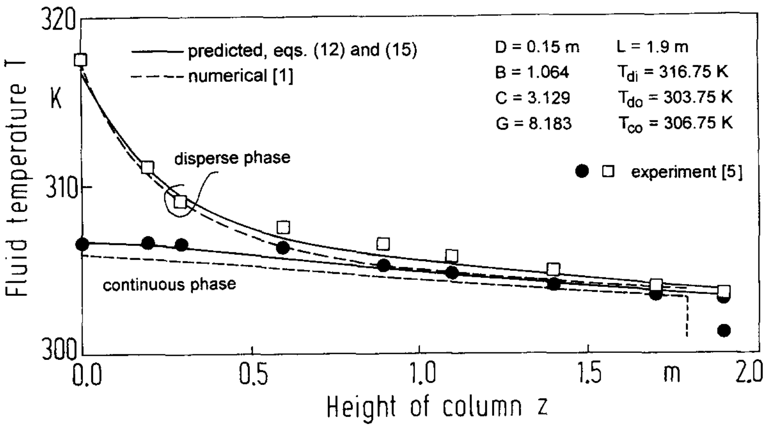

Fig. 6. Comparison of predicted temperatures with numerical [1] and experimental [5] data.

et al. [1]. The values of the parameters B, C and G given in the figures were determined from those of Battya *et al.* using the column height L as the characteristic length.

The figures show clearly that the analytical solutions agree, on average, significantly better with the experiments than the numerical results. In contrast to the numerically obtained temperatures, the analytical solution describes correctly the temperature slope of the continuous phase at the inlet, $z = 0$, of the disperse phase. The discrepancies between the analytical and the numerical temperature distributions are probably caused by the use of one different boundary condition.

CONCLUSIONS

Equations for the mean fluid temperatures in direct contact heat exchangers with countercurrent flow of continuous and disperse phase were derived by an analytical solution of the energy equations. Thermal properties of the phases, axial dispersion in the continuous phase and heat transfer conditions were considered as independent of the flow path. The constants of integration were obtained from boundary conditions specified on the basis of the fluid temperatures.

Comparisons of the analytically obtained fluid temperatures with experiments from the literature showed satisfactory agreement. The analytical solution reproduces also the slope of the temperature profile of the continuous phase at the outlet of the column. This good agreement confirms the explanations associated with the temperature gradient of the continuous phase. Therefore, the vanishing temperature gradient of the continuous phase at the column outlet frequently used in the literature [1, 2, 4] as a boundary condition (originally formulated by Danckwerts [3] for mass transfer) does not represent the physical reality.

The equations for the temperature distributions derived in this paper may also serve as a basis for the evaluation of concentration distributions in the mass transfer operations of binary systems.

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